Approach of the measurement of thermal diffusivity of mural paintings by front face photothermal radiometry

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Abstract

This paper deals with the approach of the possibilities of the front face photothermal thermography to measure the longitudinal thermal diffusivity of a mural painting. We present at first the principle of the method of measure based on the use coupled of transformed integrals and thermal quadruples. We show then the feasibility of the method by means of numerical simulation. We present then the experimental device implemented for the study. We show finally, by means of the experimental study of a sample of plaster, which the photothermal method allows in a particular case, a good estimation of the parameter longitudinal thermal diffusivity.

1. Introduction

Within the framework of the assistance to restoration of mural painting, our laboratory works since about ten year on the detection of detachment and air pocket situated in frescoes. The front face photothermal thermography, has already allowed us to detect, in situ, detachment situated in the Saint Christopher of the "Campana" collection of "Louvre", in walls painted of the church Saint Florentin of Bonnet, in ceilings painted of abbey of "Saint Savin sur Gartempe" (classified in the UNESCO world heritage) and finally in the frescoes Cocteau of the vault "Saint Pierre" of "Villefranche sur Mer" [1-3]. These qualitative studies being positive, they push us now to study the possibilities of the photothermal method in characterization of the defects. Later, the objective is to determine both the area of the defect but also the depth in which it is situated. For it we intend to proceed by an adjustment theory / experiment with the use of methods of inverse techniques of the type levenberg marquardt [4]. To feed the model evoked previously, it is necessary to know the thermophysical properties of the studied materials. Two solutions were offered to us: either appeal to bibliographical values that is to develop a method of measure of diffusivity usable in situ. For reasons of appropriate life of materials (ageing, presence of humidity, manufacturing processes, environment), it is the second solution that we chose to implement. To lead to these measures of diffusivity, we had even there two solutions: either take a sample of work of art or implement a classic method of measure of this thermophysical parameter (method flash [5]), or develop a useful method in situ and non-destructive for the studied mural painting. For reasons of good preservation of the works of art, it is this last option we chose to implement. The thickness of the couple coats - wall composing the mural painting being often of several tens centimeters, a measure of transverse thermal diffusivity is often impossible. We thus chose to develop a method of measure of longitudinal thermal diffusivity of the work of art. It is this method we present here. We present at first the principle of the method of measure, based on the use coupled of integral transformations and the quadrupole method [6]. We show then the feasibility of the method by means of numerical simulation. We present then the experimental device developed for the study. We show finally, the method gives access to a good estimation of the longitudinal diffusivity of a sample of plaster.

2. Principle of in situ measure of longitudinal thermal diffusivity

The general principle of the in situ measure of longitudinal thermal diffusivity by front face photothermal thermography is the following one : a anisotropic sample is subjected on its front face of an excitation close in time to a function delta of Dirac $\delta(t)$ and of some spatial shape $f(x,y)$. We measure then the front face field of temperature by means of a camera of infrared thermography. From the temporal evolution of this field of temperature, we calculate, by means of a mathematical post-treatment the values of thermal diffusivity of the material according to its directions of anisotropy. Let us examine in detail this mathematical post-treatment on which is based this technique of measure. λ_x, λ_y and λ_z are the thermal

conductivities of the studied sample. These thermal conductivities will be supposed constants in the time and according to the temperature (hypotheses of short analyses and weak temperature variations). ρ et c_p are the density and the calorific capacity of the same sample. The thermal diffusivities of the studied sample is a_x, a_y et a_z. The coefficients of heat exchange of rear and front faces of the sample are h_0 and h_e. e is the thickness of the material. This thickness is supposed very weak in front of the side dimensions of the sample, what allows neglecting the side heat losses of the sample. Finally, the sample is initially considered in thermal balance with its environment (fig. 1.).
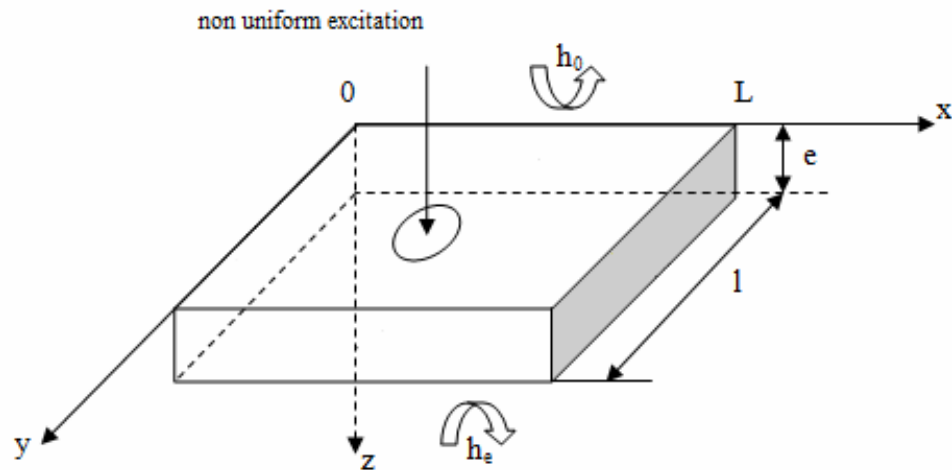

Fig. 1. *The Boundary conditions retained for the study*

The mathematical translation of these hypotheses leads to the following differential system:

$$\lambda_x (\partial^2 T / \partial x^2) + \lambda_y (\partial^2 T / \partial y^2) + \lambda_z (\partial^2 T / \partial z^2) = \rho c (\partial T / \partial t)$$

for z = 0

$$\lambda_z (\partial T / \partial z)_{z=0} = h_0 (T(z=0) - T_{ext}) - f(x, y)\delta(t)$$

for z = e

$$\lambda_z (\partial T / \partial z)_{z=0} = -h_e (T(z=e) - T_{ext})$$

for x = 0 et x = L_x,

$$\partial T / \partial x = 0$$

for y = 0 et y = L_y,

$$\partial T / \partial y = 0$$

at t = 0,

$$T = T_{ext} \qquad\qquad (1)$$

To resolve this differential system, we chose to implement three integral transformations; a Laplace transform in time associated with a cosines Fourier transform in coordinates of space x and y:

$$\theta(\alpha_n, \beta_m, z, p) = \int\limits_{t=0}^{\infty} \int\limits_{y=0}^{L_y} \int\limits_{x=0}^{L_x} T(x, y, z, t) \cos(\alpha_n x) \cos(\beta_m y) \exp(-pt) dx dy dt \qquad (2)$$

with :

- $\alpha_n = n\pi/L_x$
- $\beta_m = m\pi/L_y$

By applying this integral transformation to the previous differential system, the differential equation to be resolved in the space transformed only depends of z and can be thus resolved easily by the method of the thermal quadrupoles [7]. We obtain then:

$$\theta(\alpha_n, \beta_m, z = 0, p) = \frac{F(\alpha_n, \beta_m)(ch(\gamma_{n,m}e) + h_e sh(\gamma_{n,m}e)/(\lambda_z \gamma_{n,m}))}{\lambda_z \gamma_{n,m} sh(\gamma_{n,m}e) + (h_0 + h_e)ch(\gamma_{n,m}e) + h_0 h_e sh(\gamma_{n,m}e)/(\lambda_z \gamma_{n,m})} \qquad (3)$$

with :

- $\gamma_{n,m} = \sqrt{p/a_z + (\lambda_x/\lambda_z)\alpha_n^2 + (\lambda_y/\lambda_z)\beta_m^2}$
- $F(\alpha_n, \beta_m)$, the Fourier Laplace transform of exciting flow f(x,y) δ(t).

By taking now inverse Laplace transform of the temperature, we obtain:

$$Ln(\frac{\theta(\alpha_n, \beta_m, z = 0, t)}{\theta(0,0, z = 0, t)}) = Ln(\frac{F(\alpha_n, \beta_m)}{(F(0,0)}) - (a_x \alpha_n^2 t + a_y \beta_m^2 t) \qquad (4)$$

We notice that the longitudinal diffusivities ax and ay can be simply deducted from the slope of the curve representing the ratio of the logarithm of the coefficients of Fourier drawn with regard to time (5).

a = (slope of the curve * size of the analyzed zone) / (Fourier order ^{2} π ^{2}) (5)

3. Numerical simulation

Before switching on an experimental study, we wanted to test theoretically, this new technique of measure of longitudinal thermal diffusivity. We then began a series of simulation of the photothermal experiment. The software of resolution of the heat equation we implemented is the commercial software COMSOL. This last one, to reduce calculation time, was implemented in 2D geometry. The sample we studied in theory is a parallelepiped of plaster 6 cm in width, 2 mm in thickness and thus of 1m of length (2D geometry). The thermophysical properties taken into account are a thermal conductivity of 0.4 W.m^{-1} K^{-1}, a density of 1100 Kg.m^{-3} and finally a specific heat of 830 J.Kg^{-1} K^{-1}. These values lead to a

theoretical value of thermal diffusivity of 4, 38.10-7 m^{2}s^{-1}. The flash excitation of this sample of plaster was applied through a crenel of energy of 1000 J during 3 ms, on a 1 mm width and on a length of 1 m (2D geometry) in the center of the sample. Convective exchanges are considered on the superior and lower faces of the sample. The value of the coefficient of exchange is h=10 W.m^{-2}. K^{-1}. Finally, there also, to reduce calculation time, a progressive meshing of the sample was considered. This last one is finer at the level of the incited zone and more unrefined on the edges of the sample.

The resolution of the heat equation was made by means of the method of the finite elements. The step of used time is 0,25 seconds.

To lead in the theoretical measure of the longitudinal thermal diffusivity, we calculated at first for every step of time the thermal signature of the surface of the studied sample. The fig. 2. shows this signature initially very intense and located becomes in time more spread and less intense.
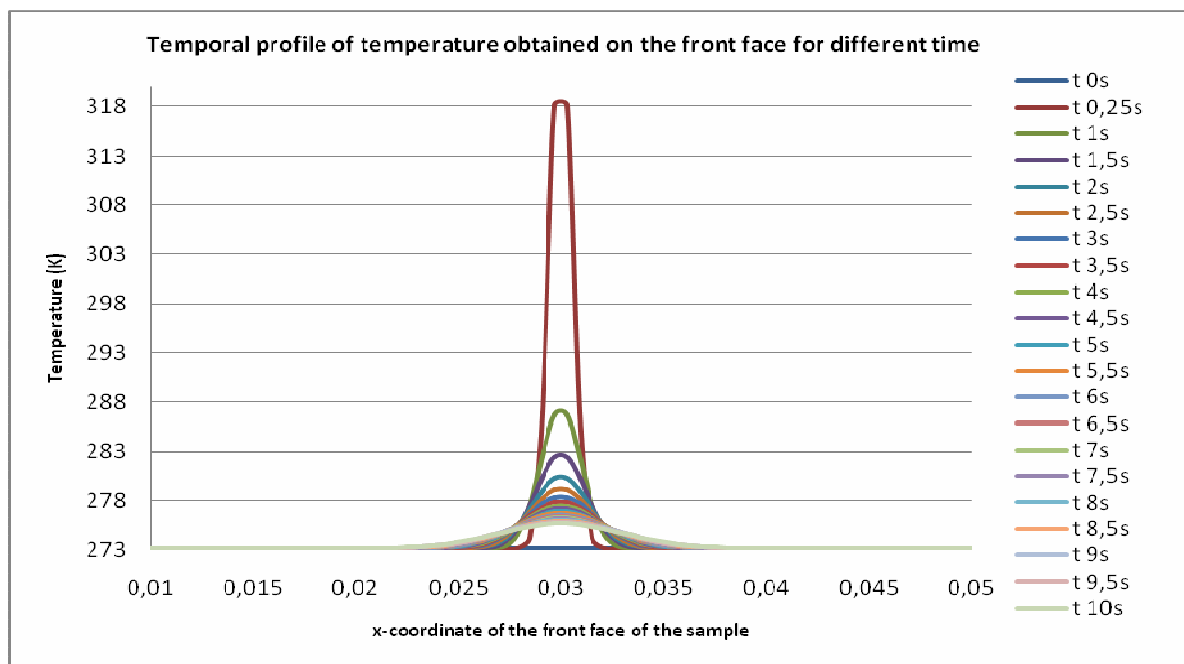

Fig. 2. *Temporal profile of temperature obtained on the front face for different time*

We calculated then the Fourier transforms in cosines of these various profiles and drew on the fig. 3, the evolution of the logarithm of the ratio of the coefficients of Fourier obtained at order 2 and at order 0. This figure shows, as the theory predicted it a line with a negative slope.
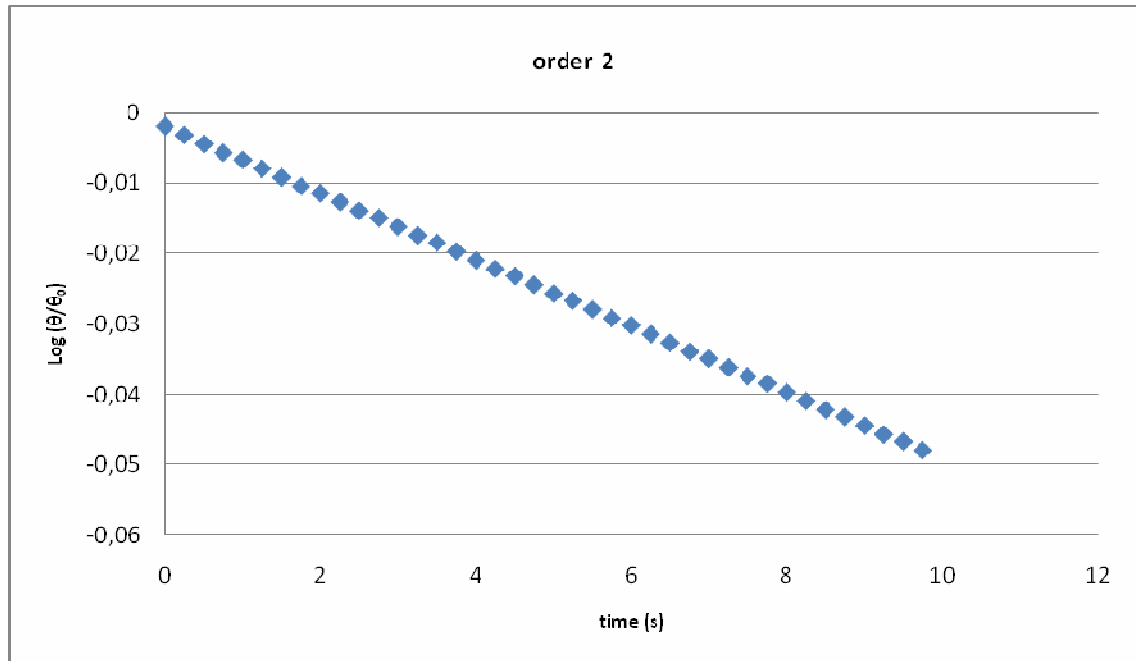
Fig. 3. *Temporal evolution of ratio of coefficients of Fourier at order 2 and order 0*

We determined finally this slope to lead to the searched thermal diffusivity. We found a value of - $4,70 \cdot 10^{-3}$ s^{-1}. As the width of the analyzed zone is 6 cm and as the order of the coefficient of Fourier considered is 2, the formula 5 leads to an estimated value of thermal diffusivityy of $4,29.10^{-7}$ m^{2}s^{-1}. This value is very close, numeric errors included, to the theoretical value equal to $4,38.10^{-7}$ m^{2}s^{-1}. It seems to show in theory the feasibility of the method.

4. Experimental device implemented

Further to the encouraging theoretical study, we decided to develop an experimental study. For that purpose, we developed a new experimental device. This one consists of an optics of excitation, of an infrared optical chain of detection and finally of a computing of piloting of the instrumentation. The source of excitation is a laser diode emitting at 810 nm associated with optics of collimation and focusing. The optics of infrared acquisition is constituted by a "long waves" camera using bolometers detectors, working in macro mode (to have a sufficient spatial resolution). This last one is perpendicularly placed in the sample, in the distance of about 5 cm. The laser beam is, because of the dimensions of the camera; send in a tilted way on the sample to be analyzed. Its shape is slightly elliptic. The diode laser is piloted to make it emit a power of 2 w during 20 ms. The frequency of acquisition of the camera of infrared thermography is 50Hz.
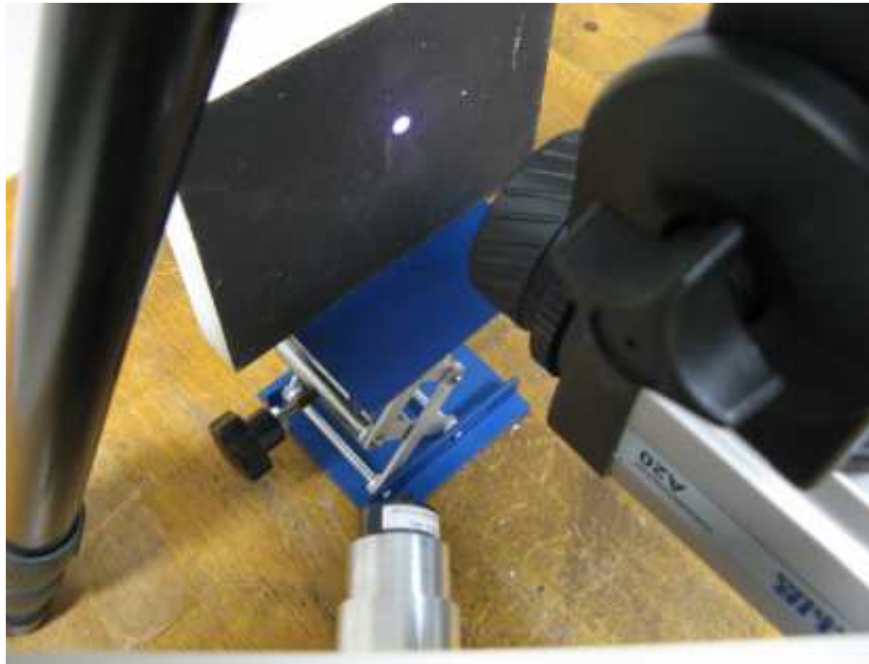
Fig. 4. *The eexperimental device implemented*

5. Sample analyzed in an experimental way

The sample we analyzed experimentally in this study is block of plaster of 12 cm * 15 cm dimension and 2,2 cm thick. It is covered with a thin black coating of paint on the analyzed side, to simulate the presence of a pictorial layer. At first, to determine exactly its thermal diffusivity, we studied it at first with a flash diffusivimeter of reference of the LEMTA of Nancy (France, hypothesis of isotropic thermal diffusivity). For that purpose, we cut a part of the sample and manufactured it to reduce its thickness to 6,19 mm, to make it compatible with a measure with this diffusivimeter. The duration of excitation was 5 ms. The measure of temperature, made at the rear face of the sample was assured by a semi conductor thermocouple of type tellurium of bismuth .Three classic modes of analysis of the obtained thermograms were implemented, the method of partial-times, the method of temporal moments and an adjustment theory / experiment. The values of thermal diffusivity obtained are: $3,49 \ 10^{-7} \ m^2$ / by means of the method of partial-times, $3,53 \ 10^{-7} \ m^2$/, by means of the method of temporal moments and $3,49 \ 10^{-7} \ m^2$ / by means of an adjustment theory / experiment (fig. 5.).
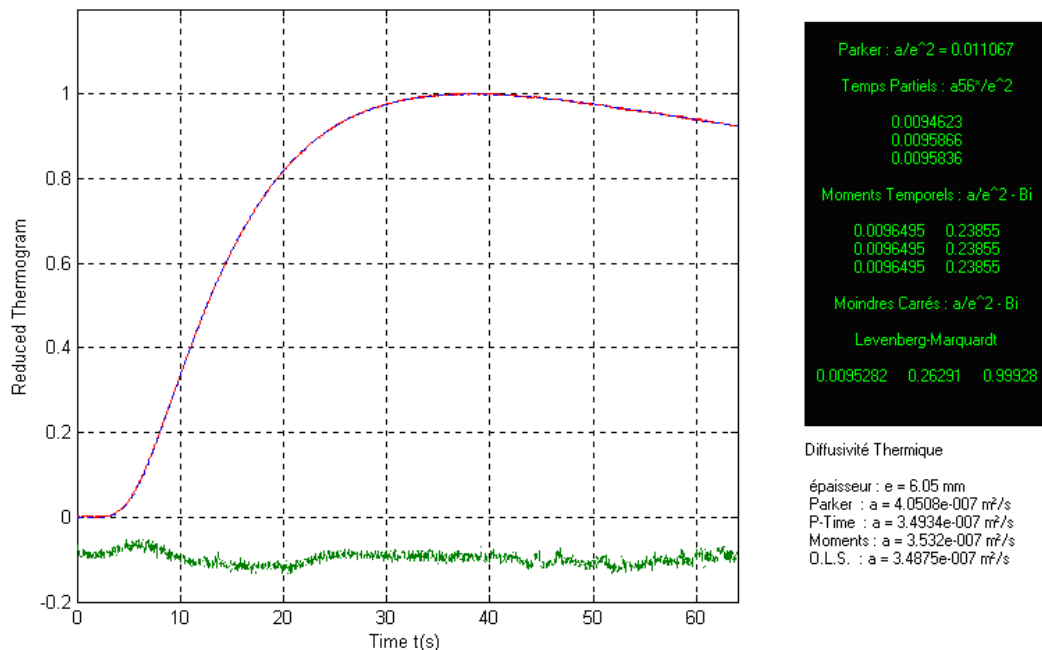
Fig. 5. *Measurement of the thermal diffusivity of the analyzed sample by the classics methods (partial-times method, temporal moments method and adjustment theory / experiment method).*

6. Experimental results obtained

The part of the not cut sample was then analyzed by front face photothermal method. As we mentioned it previously, this sample was thus enlightened on a surface about 1 mm^2, with a power of 2 watt during 20 ms. This heating was filmed in a synchronous way with the excitation by a camera of infrared thermography type Flir A20, implemented in a macro mode. On the fig. 6., we present four representative infrared images of the temporal evolution of the thermal signature of the laser spot. We indeed notice that this last one becomes more spread and less intense with time.
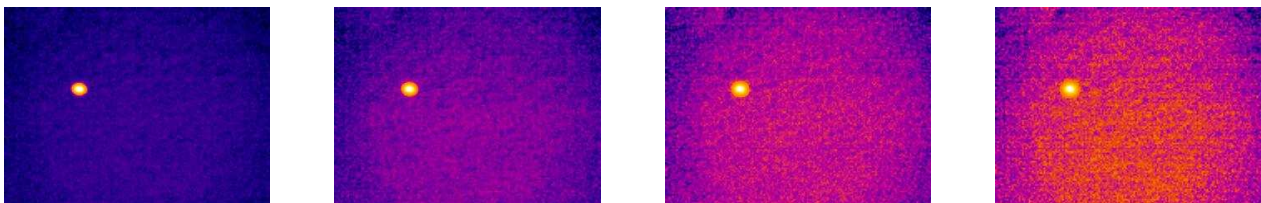

Fig. 6. *Temporal evolution of the infrared signature of the heating*

As the simulations and as plans it the theory, we took cosine Fourier transforms of these various images. We then calculated, for each of them the logarithm of the ratio of the coefficients of Fourier obtained at order 2 at of the coefficients of Fourier obtained at order 0. The fig. 7. represents the result obtained. It shows, as the theory plans it, a line of negative slope. It is equal, in this case to 2.558 s^{-1}.
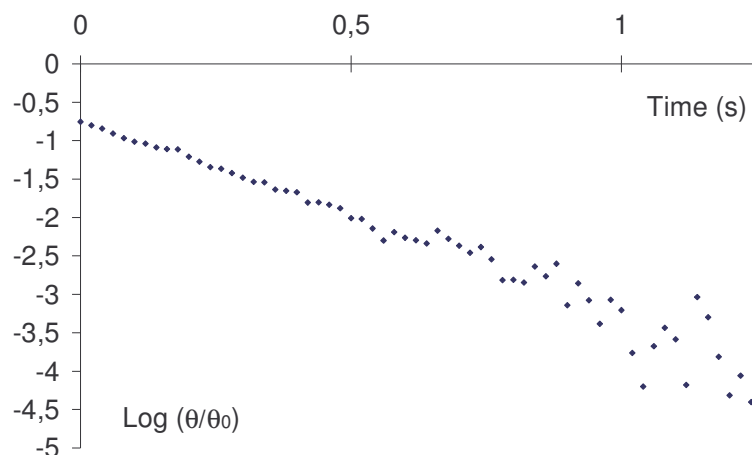
Fig. 7. *Evolution of the ratio of logarithm of the coefficients of Fourier obtained at order 2, c calculated for each infrared image.*

Finally, to succeed, in a measure of longitudinal diffusivity, we calibrated spatially our experimental device. For that purpose, we placed a calibrated surface on the studied object and deducted from the infrared image obtained, the spatial dimension seen by pixel. We found a value of 116 µm. The formula 5, then allowed us to determine the longitudinal value of the thermal diffusivity of the analyzed sample of plaster. We find a mean value equal to 3,49 10^{-7} m^2s^{-1}. This value is very close to those obtained classically with the flash method, what seems to show the possibilities of the method in measure in situ, of this thermophysical parameter.

7. Conclusion

In this work, we tried to approach in an experimental way, the possibilities of the photothermal thermography in measure in situ of longitudinal thermal diffusivity of mural painting.

We presented at first the principle of the method of measure.

We then showed the feasibility of the method by means of numerical simulation.

We presented in a third time, the experimental device implemented for the study.

We finally showed, by means of the experimental study of a sample of plaster that the photothermal method allows in a particular case a good estimation of the parameter thermal diffusivity.

This experimental result, obtained on a particular sample, is encouraging because seeming to open the way to the photothermal characterization in situ of works of art. It asks now to be generalized. Studies going to this direction are in progress.

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